Reply[4] by A. Vishen, G. S. Singh, and F. E. Gardiol[5]

We would like to thank Dr. Fikioris, Dr. Roumeliotis, and Dr. Davidovitz for their interest and comments on our paper. We welcome this opportunity to clarify some aspects and hope in this way to clear up some confusion.

The cutoff wavenumbers, which are characterized by $\beta = 0$, have been considered in our paper [1], as is indicated by its title. This implies that in this work $k = k_c$, so that the error which Fikioris and Roumeliotis claim to have located just does not exist.

It seems that the basic spirit behind the approach discussed in [2] and used in our paper has been overlooked, resulting in comments on our statement "a rigorous mathematical derivation". The use of the addition theorems for the Bessel functions in the context of a solution to the Helmholtz equation is certainly not new; the same had been used some 40 years ago while studying eccentric control-rod problems in thermal nuclear reactors [3]. The Fourier expansions which we have made use of do provide a more general technique for solving problems with complicated boundaries. In fact, Nagaya and coworkers [4, and references therein] have already applied this approach in studying the vibration of membranes and plates of various geometries. However, for the geometry under discussion, our approach leads to the infinite set of linear equations identical with those given in [5] and [6].

One finds from the tables in [5] that symmetric and antisymmetric modes of a given higher order have the same value of g_{nm} up to the eighth decimal place. This gives a degeneracy which is simply an artifact of an approximate but algebraically tedious approach. The authors of [5] have not developed a form which would support their claim for the onset of bifurcation by inclusion of the fourth-order terms. On the other hand, they state that "the agreement for $k_c d < 1$ (sometimes even $k_c d > 1$) is remarkable." In contrast, we find that when $g_{nm} \simeq -6$ [5, tables II and III], the cutoff wavenumber goes to zero for $k_c d \simeq 0.4$ and even becomes negative for $k_c d \gtrsim 0.4$. This is clearly meaningless, referred to in [1] as "nonphysical."

One may be able to circumvent the bizarre situation if one considers the work in [5] to have a mode-dependent range of validity. But then this work loses its significance. One may even question the wisdom of using an approximate method whose validity for each mode requires that it be checked by comparison with experimental or exact theoretical work like ours. Of course, the numerical resolution in our work requires repetitive calculations. But this is hardly relevant nowadays when ample computer power is available. Furthermore, one does not have to worry over the validity of results for any particular value of $k_c d$.

It was noted in [1] that some of the cutoff values obtained lie outside (but near) the bounds reported by Kuttler [7]. Also, some of the values obtained by the approximate methods could not (as yet) be obtained with the rigorous approach. These two kinds of discrepancies are still unresolved; the comments by Fikioris and Roumeliotis do not provide any new insight on these issues.

[4]Manuscript received December 16, 1986.
[5]A. Vishen is with the Department of Physics and Astrophysics, University of Delhi, Delhi 110 007 India.
G. S. Singh and F. E. Gardiol are with the Laboratoire d'Electromagnetisme et d'Acoustique, Chemin de Bellerive 16, CH-1007 Lausanne, Switzerland.
IEEE Log Number 8613295.

References

[1] A. Vishen, G. P. Srivastava, G. S. Singh, and F. Gardiol, "Calculation of cutoff wavenumbers for TE and TM modes in tubular lines with offset center conductor," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-34, pp. 292–294, Feb 1986.

[2] G. S. Singh and L. S. Kothari, "On the solution of the two-dimensional Helmholtz equation," *J Math. Phys.*, vol. 25, pp. 810–811, Apr 1984.

[3] L. W. Nordheim and R. Scalettar, "Theory of pile control rods," *Phys. Rev.*, vol. 70, p. 115, July 1946.

[4] K Nagaya and Y. Hai, "Free transverse vibration of composite membranes with arbitrary shape," *J. Sound Vibration*, vol. 100, pp. 123–134, May 1985

[5] J. A. Roumeliotis, A. B. M. Siddique Hossain, and J. G. Fikioris, "Cutoff wavenumbers of eccentric circular and concentric circular–elliptical metallic waveguides," *Radio Sci.*, vol. 15, pp. 923–937, Sept. 1980.

[6] G. I. Veselov and S G. Semenov, "Theory of circular waveguide with eccentrically placed metallic conductor," *Radio Eng. Electron. Phys*, vol 15, pp. 687–690, Apr. 1970.

[7] J. R Kuttler, "A new method for calculating TE and TM cutoff frequencies of uniform waveguides with lunar or eccentric annular cross section," *IEEE Trans. Microwave Theory Tech.*, vol MTT-32, pp. 348–354, Apr 1984

Comments on "Millimetric Nonreciprocal Coupled-Slot Finline Components"

E. JENSEN AND C. SCHIEBLICH

Abstract — **A statement is made concerning the feasibility of nonreciprocal components with ferrites magnetized in the direction of propagation.**

In the paper in question,[1] an isolator is presented operating with a ferrite magnetized in the direction of propagation inside waveguide with constant cross section.

Since only reciprocal devices with these properties have become known until now (Reggia–Spencer phase shifter), some general remarks should be made concerning the features of devices with gyrotropic media.

Let us consider an arbitrary waveguide section W (Fig. 1) between the planes A_1 and A_2, which are in the xy plane of a Cartesian coordinate system. The waveguide is completely or partly filled with gyromagnetic material with the permeability tensor

$$\vec{\mu} = \mu_0 \begin{pmatrix} \mu & -j\kappa & 0 \\ j\kappa & \mu & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (1)$$

i.e., the magnetic bias field is applied in $+z$ direction. At A_1, we assume an arbitrary transversal field distribution $\vec{E}_1(x, y)$, $\vec{H}_1(x, y)$. This yields a field distribution $\vec{E}_2(x, y)$, $\vec{H}_2(x, y)$ at A_2.

Inserting a magnetic wall M in the xy plane and applying image theory yields the waveguide section W' with the field

Manuscript received October 8, 1986.
The authors are with the Technische Universität Hamburg-Harburg, Arbeitsbereich Hochfrequenztechnik, D-2100 Hamburg 90, West Germany.
IEEE Log Number 8613292.
[1]L. E. Davis and D B. Sillars, *IEEE Trans Microwave Theory Tech.*, vol MTT-34, pp 804–808, July 1986.

0018-9480/87/0400-0470$01.00

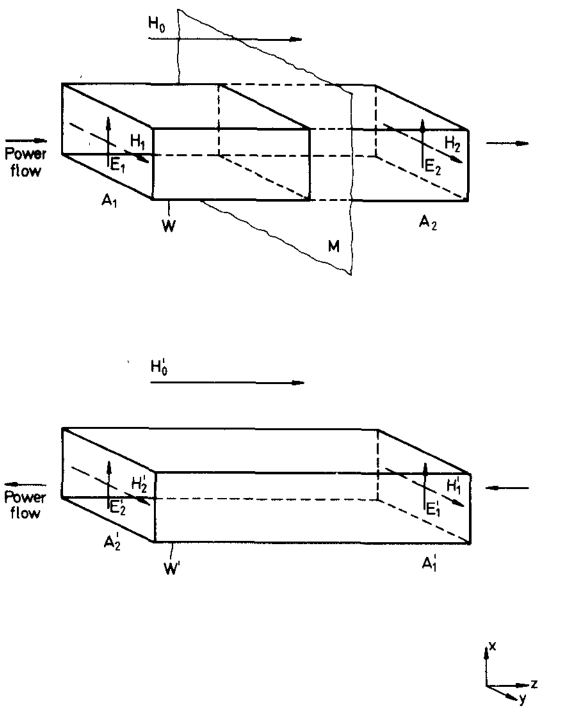

Fig. 1. Waveguide section.

distributions $\vec{E}_1'$, $\vec{H}_1'$ and $\vec{E}_2'$, $\vec{H}_2'$ at the planes A_1' and A_2', respectively. The following relations hold for the transversal field components:

$$\vec{E}_i'(x,y) = \vec{E}_i(x,y)$$

$$\vec{H}_i'(x,y) = -\vec{H}_i(x,y) \tag{2}$$

where $i = 1,2$. Field components polarized circularly in the xy plane keep their sense of rotation, so that the permeability tensor (1) remains unchanged; hence, the bias field is still in $+z$ direction.

Integrating the Poynting vector $\vec{E} \times \vec{H}$ across the boundary planes reveals that the power flow through A_1 in the $+z$ direction equals that through A_1' in the $-z$ direction, and the same holds for the planes A_2 and A_2'.

If the waveguide section W exhibits reflection symmetry with respect to the midplane between A_1 and A_2, W and W' differ only in the direction of bias field related to the direction of propagation. In both cases, the ratio of outgoing to incident power is the same; i.e., the device is reciprocal.

This statement is still valid for devices with more than two ports, provided that all reference planes are in the xy plane. Bias fields parallel to the reflection plane, i.e., transverse to the direction of propagation, must be reversed in the mapping procedure because field components polarized circularly in a plane orthogonal to the reflection plane change their sense. In this case, the above considerations result in the generalized reciprocity theorem that simultaneous exchange of ports and bias field direction does not alter the transmission properties [1].

The devices presented by Davis and Sillars were biased by a bar magnet alongside the ferrite region. This may give rise to transversely biased field components. It should be pointed out here that for devices with the above-mentioned symmetrical properties, only the transversely biased field components render possible nonreciprocal behavior.

Reply[2] *by L. E. Davis and D. B. Sillars*[3]

We would like to thank the above authors for their interest in our paper and for the questions related to the isolators described therein. They made two principal points: (a) that the ferrite is magnetized in the direction of wave propagation and the component should be reciprocal in accordance with the generalized reciprocity theorem and (b) that the steady magnetic bias field was applied with permanent magnets alongside the ferrite region and these may have caused stray transverse static field components which could have caused the nonreciprocal behavior.

Taking the second point first, the permanent magnets could cause small transverse static field components in the ferrite, but we believe these would have a negligible effect. Experiments with a transverse bias field early in the investigation showed that it did not cause any significant nonreciprocity in this coupled-mode structure at 26–40 GHz with the magnitudes of field available to us. Also, it may be noted that the values of bias field used in these axially biased components are very much smaller than have been used in transversely biased field-displacement or resonance components and therefore stray transverse fields would not be expected to cause appreciable nonreciprocity. To minimize transverse-field effects, some experiments were performed with the structure aligned coaxially at the center of a solenoid and it was found that a differential insertion loss of the order of 12 dB could readily be obtained.

Regarding the first point, it is the inhomogeneity and asymmetry of the structure that cause the nonreciprocal behavior of the odd mode.

The ferrite is placed only on one side of the finline double-slot septum, and with a fixed direction of propagation the nonreciprocity can be demonstrated either by reversing the applied field H_0 or by placing the ferrite slab on the other side of the septum. The nonreciprocity takes the form of an RF field displacement away from one slot towards the other when the "quasi-odd mode" is launched in the coupled slots. (In the absence of H_0, the odd normal mode is defined in Fig. 2(b) of our paper.) Using this behavior, isolators were realized with structures that do not have magnetic-wall reflection symmetry. For example, the isolator shown in Fig. 6 of our paper does not look the same at port 1 and port 2. Looking in from the left, the ferrite is on the right of the septum and H_0 is antiparallel to the direction of propagation; looking in from the right, H_0 is again antiparallel but the ferrite is on the left. With the isolator shown in Fig. 8 of our paper, looking from the left, ferrite (1) is on the right and H_0 is antiparallel. Looking from the right, ferrite (2) is again on the right but H_0 is parallel with the direction of propagation.

Therefore, these axially magnetized isolators do not violate the generalized reciprocity theorem.

References

[1] R. F. Harrington and A. T. Villeneuve, "Reciprocity relationship for gyrotropic media," *IRE Trans. Microwave Theory Tech.*, vol. MTT-6, pp. 308–310, 1958.

[2] Manuscript received December 13, 1986.
[3] The authors are with the Department of Electrical and Electronic Engineering, Paisley College of Technology, Paisley PA1 2BE, Scotland.
IEEE Log Number 8613293.